Justify using First Derivative Test or Second Derivative Test.

1. A printed page will have margins of 2 cm at the top and sides and 4 cm at the bottom. If the printed area is 150cm², find the dimensions of the whole page so that its area will be a minimum.

Let \( x \) be the length of the page in cm.

\[
\begin{align*}
2y &= 150 \\
\Rightarrow y &= \frac{150}{x}
\end{align*}
\]

Minimize area

\[
\text{Area} = (x + 4)(y + 4)
\]

\[
A(x) = (x + 4)(\frac{150}{x} + 6)
\]

\[
A'(x) = \frac{6(x^2 - 100)}{x^2}
\]

Set \( A'(x) = 0 \)

\[
x = 10 \text{ cm}
\]

\[
A''(x) = \frac{1200}{x^3}
\]

\[
A''(10) = \frac{3}{5}
\]

Pos, concup, rel min.

Dimensions are 14 cm \( \times \) 21 cm

2. Painters are painting the second floor exterior wall of the building that adjoins a busy sidewalk.

A corridor 2m wide and 3 m high is built to protect pedestrians. What is the length of the shortest ladder that will reach from the ground over the corridor of the wall of the building?

\[
\begin{align*}
\text{minimize } z \\
2 &= \sqrt{(x+2)^2 + y^2}
\end{align*}
\]

\[
\frac{x}{3} = \frac{x+2}{y}
\]

\[
x, y = (x+2)
\]

\[
y = \frac{3(x+2)}{x}
\]

\[
z'(x) = \frac{1}{x} \sqrt{x^2 + 9} + \left( \frac{1}{\sqrt{x^2 + 9}} - \frac{\sqrt{x^2 + 9}}{x^2} \right) (x+2)
\]

Solve \( z'(x) = 0 \)

\[
x \approx 2.621 \Rightarrow \text{state a}
\]

\[
z \approx 7.023 \text{ m}
\]

\[z''(2.621) = 2.75, \] concup, rel min
3. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3. Justify your answer.

\[ x^2 + y^2 = 9 \quad \rightarrow \quad x^2 = 9 - y^2 \]

Maximize volume

\[ V = \frac{\pi}{3} x^2 (3+y) \]

\[ V = \frac{\pi}{3} (9-y^2)(3+y) \]

\[ V'(y) = -\pi (y^2 + 2y - 3) \]

Set \( V'(y) = 0 \)

\[ y = 1 \quad \text{or} \quad y = -3 \]

\[ V''(y) = -2\pi (y+1) \]

\[ V''(1) = -4\pi \quad \text{min, conc. down, all max} \]

\[ V = \frac{\pi}{3} \cdot 8(y) \]

\[ x = \sqrt{8} \]

Max volume is \( \frac{32\pi}{3} \)

4. Two posts, one 15 ft high and one 25 ft high, stand 38 ft apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least wire? Justify.

\[ W(x) = \sqrt{x^2 + 225} + \sqrt{625 + (38-x)^2} \]

\[ W'(x) = \frac{x-38}{\sqrt{x^2 - 76x + 2069}} + \frac{x}{\sqrt{x^2 + 225}} \]

Set \( W'(x) = 0 \)

\[ x = \frac{57}{4} \quad \text{CP (14.25)} \]

\[ x = 14.25 \quad \text{all min} \]

1st Deriv. Test

Place stake 14.25 ft from 15 ft stake.

Optimization Quiz tomorrow (Wed, Jan 6)

TEST on Fri, Jan 8th.