2) \( y = -2x^3 + 6x^2 - 3 \) (Identify any Local or Absolute Extrema)

\[ f(x) = -6x^2 + 12x \]

Sign of \( f' \):

\[ f' = - + - \]

\( -6x(x-2) = 0 \)

Critical Points: \( x = 0 \) or \( x = 2 \)

\( f(0) = -3 \)

(“careful - the \(-6\) impacts the sign of \( f' \))

By First Deriv. Test,

Local Max at \((2, 5)\)

Local Min at \((0, -3)\)

3) \( y = 2x^4 - 4x^2 + 1 \)

Sign of \( f' \):

\[ f'(x) = 8x^3 - 8x \]

\[ 8x(x^2-1) = 0 \]

\[ f(0) = 1 \]

By First Deriv. Test,

Local Min at \((-1, -1)\) and \((1, -1)\)

Local Max at \((0, 1)\)

Abs Min at \((-1, -1)\) and \((1, -1)\)

Find where the function is concave up and down

4) \( y = 4x^3 + 21x^2 + 36x - 20 \)

\[ f'(x) = 12x^2 + 42x + 36 \]

\[ f''(x) = 24x + 42 \]

\[ 12x^2 + 42x + 36 = 0 \]

\[ 2x^2 + 7x + 6 = 0 \]

\[ (2x + 3)(x + 2) = 0 \]

Critical Points: \( x = -\frac{3}{2} \) or \( x = -2 \)

\( f''(x) = - + + \)

\( x = -\frac{3}{2} \) or \( x = -2 \) are points of

\[ f(x) \] concave down \(-\frac{3}{2}, -2\) because \( f''(x) < 0 \)

\( f(x) \) concave up \((-\frac{3}{2}, -2)\) because \( f''(x) > 0 \)

\( f(x) \) concave down \((-2, \infty)\)

\( f(x) \) concave up \((-\infty, -2)\)
8) \[ y = -x^4 + 4x^3 - 4x + 1 \]
\[ f'(x) = -4x^3 + 12x^2 - 4 \]
\[ f''(x) = -12x^2 + 24x \]
\[ -12x^2 + 24x = 0 \]
\[ -12x(x-2) = 0 \]
Possible Points: \( x = 0 \) or \( x = 2 \)
\[ \text{Sign of } f'' = - + - \quad (\text{Notice the } -12 \text{ has an impact}) \]
At:
concave up concave down
\[ f(x) \text{ is concave up on } (0, 2) \text{ because } f''(x) > 0 \]
\[ f(x) \text{ is concave down on } (-\infty, 0) \cup (2, \infty) \text{ because } f''(x) < 0 \]

Find all points of inflection:

13) \[ y = xe^x \]
\[ f'(x) = xe^x + e^x \]
\[ f''(x) = xe^x + e^x + e^x = xe^x + 2e^x = e^x(x+2) \]
\[ e^x(x+2) = 0 \]
\[ x = -2 \quad \text{possible point} \]
\[ f(-2) = -2e^{-2} \]
\[ \text{point at } (-2, -2e^{-2}) \text{ because } f''(x) = 0 \]
\[ \text{and concavity changes} \]

18) \[ y = x^{1/2}(x+3) \]
\[ f'(x) = \frac{1}{2} x^{-1/2} + (x+3) \cdot \frac{1}{2} x^{-1/2} \]
\[ f''(x) = \frac{1}{2} x^{-1/2} + \frac{1}{2} \left[ (x+3)(-1/2)x^{-3/2} + x^{-1/2} \right] \]
\[ f''(x) = \frac{1}{2} x^{-1/2} - \frac{1}{4} (x+3)x^{-3/2} + \frac{1}{2} x^{-1/2} \]
\[ = \frac{1}{2} x^{-1/2} - \frac{1}{4} (x+3)x^{-3/2} \]
\[ = \frac{1}{4} x^{-3/2} \left[ -4x + (x+3) \right] \]
Domain for this function is $X \geq 0$ (because of $x^{1/2}$)

$$f''(x) = -\frac{1}{4} x^{-3/2} (-3x+3) = -\frac{1}{4} (-3) x^{-3/2} (x-1)$$
$$= \frac{3}{4} x^{-3/2} (x-1) = \frac{3(x-1)}{4 \sqrt{x^3}}$$

$f''(x) = 0$ when $x = 1$

Oops! $x = 0$ is not included in the domain of $f''$ (also pass poi)

pois at $x = 0$ and $x = 1$

because $f''(1) = 0$ and

$f''(0)$ is undefined, and concavity changes at concavity changes

pois to $x = 0$ and $x = 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 2$</td>
<td>falling, concave up</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>horizontal tangent</td>
</tr>
<tr>
<td>$2 &lt; x &lt; 4$</td>
<td>rising, concave up</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>poi</td>
</tr>
<tr>
<td>$4 &lt; x &lt; 6$</td>
<td>rising, concave down</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>horizontal tangent</td>
</tr>
<tr>
<td>$x &gt; 6$</td>
<td>falling, concave down</td>
<td></td>
</tr>
</tbody>
</table>
51) (a) Find Abs Extrema of $f$ and where they occur.
(b) Find any p oi
(c) Sketch a possible graph of $f$

$f$ is continuous on $[0, 3]$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$0 &lt; x &lt; 1$</th>
<th>$1 &lt; x &lt; 2$</th>
<th>$2 &lt; x &lt; 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$f'$</td>
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<td>0</td>
<td>DNE</td>
<td>-3</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f''$</td>
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<td>DNE</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Abs Max at $(1, 2)$ Abs Min at $(3, -2)$
(b) no p oi, (concavity does not change)
(e) see above