#1 - #2
A certain calculus student (who shall remain nameless for his own protection) hit Mrs. Harlow in the head with a huge snowball. If the snowball was melting at the rate of 10 cubic feet per minute, at what rate was the radius changing when the snowball was 1 foot in radius (Problem #1).

At what rate was the radius changing when the snowball was 2 feet in radius (Problem #2)?

\[ V = \frac{4}{3} \pi r^3 \]
\[ \frac{dV}{dt} = \frac{4}{3} \pi \cdot 3r^2 \frac{dr}{dt} \]
\[ \frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt} \]
\[ r = 1 \]
\[ -10 = 4 \pi \cdot 1 \cdot \frac{dr}{dt} \]
\[ \frac{dr}{dt} = \frac{-10}{4 \pi} = \frac{-5}{2 \pi} \text{ ft/min} \]

#3 - #4
A baseball diamond is 90 feet square. Coach Steele runs from first base to second base at the unbelievable speed of 25 feet per second. How fast is he moving away from home plate when he is 30 feet from first base (Problem #3)? Find \( \frac{dx}{dt} \) when \( x = 30 \)

How fast is he moving away from home plate when he is 45 feet from first base (Problem #4)?

\[ x^2 + y^2 = 900 \]
\[ x = 30 \]
\[ y = 30 \]
\[ \frac{dx}{dt} = 25 \text{ ft/sec} \]

\[ \frac{dy}{dt} = \frac{25 \sqrt{10}}{10} = \frac{5 \sqrt{10}}{2} \text{ ft/sec} \]

#5
Water flows at 8 cubic feet per minute into a cylinder with radius 4 feet. How fast is the water level rising when the water is 2 feet high?

\[ V = \pi r^2 h = 16 \pi h \]
\[ \frac{dV}{dt} = 16 \pi \frac{dh}{dt} \]
\[ 8 = 16 \pi \frac{dh}{dt} \]
\[ \frac{dh}{dt} = \frac{1}{2} \text{ ft/min} \]
The DSF soaking pool is an inverted cone with height 20 meters and radius 5 meters. It is being filled by Coach Brinkley with a hose which pumps in water at the rate of 3 cubic meters per minute. When the water level is 2 meters, how fast is the water level rising (Problem #6)?

How fast is the radius changing at this moment (Problem #7)?

\[
\frac{dv}{dt} = \frac{\pi}{3} \left( \frac{h}{4} \right)^2 \cdot 2 \cdot \frac{h}{3} = \frac{\pi h^3}{48} \quad \frac{v}{h} = \frac{5h}{20} = \frac{h}{4}
\]

Find \( \frac{dh}{dt} \) when \( h = 2 \)

\[
\frac{dv}{dt} = \frac{\pi h^3}{48} \quad \frac{dh}{dt} = \frac{12}{\pi} \quad \frac{dr}{dt} = \frac{1}{4} (\frac{12}{\pi}) = \frac{3}{11} \text{ m/min}
\]

A stone is dropped into the James River, causing circular ripples whose radii increase by 2 meters/second. How fast is the disturbed area growing when the outer ripple has radius 5 meters (Problem #8)?

How fast is the radius increasing at that moment (Problem #9)?

\[
A = \pi r^2 \quad r = 5 \quad \frac{dr}{dt} = 2 \text{ m/sec}
\]

Find \( \frac{dA}{dt} \)

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi (5) (2) = 20\pi \text{ m}^2/\text{sec}
\]

A fish is being reeled in at a rate of 2 meters/second by a fisherman at Deep Run Park. If the fisherman is sitting on the dock 30 meters above the water, how fast is the fish moving through the water when the line is 50 meters long (Problem #10)?

How fast is the fish moving when the line is only 31 meters (Problem #11)?

\[
x^2 + 30^2 = x^2 + 1600 \quad x = 40
\]

\[
\frac{dx}{dt} = \frac{x^2 + 30^2 - 31^2}{2 \cdot 30} = \frac{2x}{2} = \frac{2\sqrt{61}}{2} = \sqrt{61}
\]

\[
\frac{dx}{dt} = \frac{-62}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = -62 \text{ m/sec}
\]
#12
A student at DSF was painting Mrs. Harlow's trailer (no, we're not moving back!) while standing at the top of a 25-foot ladder. She was horrified to discover that the ladder began sliding away from the base of the trailer at a constant rate of 2 feet per second. At what rate was the top of the ladder carrying the student toward the ground when the base of the ladder was 17 feet away from the "learning cottage"?

\[ x^2 + y^2 = 25^2 \]
\[ 17^2 + y^2 = 25^2 \]
\[ y^2 = 336 \]
\[ y = \sqrt{4 \cdot 84} = \sqrt{4 \cdot 21} = 4 \sqrt{21} \]
\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]
\[ 2 \cdot 17 \cdot \frac{dy}{dt} + 4 \sqrt{21} \cdot \frac{dy}{dt} = 0 \]
\[ 24 = -4 \sqrt{21} \frac{dy}{dt} \]
\[ \frac{dy}{dt} = -\frac{34}{4 \sqrt{21}} = -\frac{17 \sqrt{21}}{2 \sqrt{21}} \]
\[ \frac{dy}{dt} = -\frac{17}{4} \text{ ft/sec} \]

#13
A spherical balloon was losing air at the rate of 5 cubic inches per second. At what rate was the radius of the balloon decreasing when the radius was 5 inches?

\[ V = \frac{4}{3} \pi r^3 \]
\[ \frac{dV}{dt} = \frac{4}{3} \pi \cdot 3r^2 \frac{dr}{dt} \]
\[ \frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt} \]
\[ -5 = 4 \pi (25) \frac{dr}{dt} \]
\[ \frac{dr}{dt} = -\frac{1}{20} \text{ inches/sec} \]

#14
Oil spills into Echo Lake in a circular pattern. If the radius of the circle increases at a constant rate of 3 feet per minute, how fast is the area of the spill increasing at the end of 10 minutes?

\[ \frac{dr}{dt} = 3 \text{ ft/min} \]
\[ \text{after 10 mins, } r = 30 \text{ ft} \]
\[ A = \pi r^2 \]
\[ \frac{dA}{dt} = 2 \pi r \frac{dr}{dt} \]
\[ \frac{dA}{dt} = 2 \pi (30) \cdot 3 \text{ ft/min} \]
\[ \frac{dA}{dt} = 180 \pi \text{ ft}^2 \text{/min} \]
Coffee is draining from a conical filter, diameter and height both 6 inches, into a cylindrical coffee pot, diameter also 6 inches. The rate at which coffee drains from the filter into the pot is 10 in³/min.

How fast is the level in the pot rising when the coffee in the cone is 5 in. deep? (Problem #15)

How fast is the level of coffee in the cone falling at that moment? (Problem #16)

\[ V = \frac{1}{3} \pi r^2 h \]

\[ h = 6 \quad \Rightarrow \quad r = \frac{h}{6} = \frac{3}{2} \quad r = \frac{h}{2} \]

\[ V = \frac{\pi}{3} \left( \frac{h}{2} \right)^2 \cdot h \]

\[ \frac{dV}{dt} = \frac{\pi}{12} \cdot 3 h^2 \frac{dh}{dt} \]

\[ \frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt} \]

\[-10 = \frac{\pi}{4} \cdot (5)^2 \cdot \frac{dh}{dt} \]

\[ \frac{-10}{4} = \frac{dh}{dt} \]

\[ \frac{dh}{dt} = -\frac{8}{5 \pi} \frac{\text{in}}{\text{min}} \]

\[ V = \frac{\pi}{3} r^2 h \]

\[ d = 6 \quad \Rightarrow \quad r = 3 \]

\[ V = \frac{\pi}{3} (3)^2 \cdot h \]

\[ \frac{dV}{dt} = 9 \pi \frac{dh}{dt} \]

\[ + 10 = 9 \pi \frac{dh}{dt} \]

\[ \frac{dh}{dt} = \frac{10}{9 \pi} \frac{\text{in}}{\text{min}} \]