Differentiate each of the following with respect to time \((t)\).
In other words, treat all variables as functions of time.

ex - Look at the Pythagorean relationship: \(x^2 + y^2 = r^2\).
If no quantity remains constant, think of this relationship as \([x(t)]^2 + [y(t)]^2 = [r(t)]^2\).
Differentiating implicitly, you would get:

\[
2 \cdot x(t) \cdot \frac{dx}{dt} + 2 \cdot y(t) \cdot \frac{dy}{dt} = 2 \cdot r(t) \cdot \frac{dr}{dt}.
\]

ex - Try another one: \(\frac{a}{b} = c\) Differentiating with respect to time \((t)\), you would get:

\[
\frac{b \cdot \frac{da}{dt} - a \cdot \frac{db}{dt}}{b^2} = \frac{dc}{dt}.
\]

Use these familiar formulas to differentiate with respect to time.

1. \(V = \frac{4}{3} \pi \cdot r^3\) **Vol. of Sphere**
\[
\frac{dV}{dt} = \frac{4}{3} \pi \cdot 3 \cdot r^2 \cdot \frac{dr}{dt}
\]

2. \(P = 2l + 2w\) **Perimeter of Rectangle**
\[
\frac{dP}{dt} = 2 \cdot \frac{dl}{dt} + 2 \cdot \frac{dw}{dt}
\]

3. \(A = \frac{1}{2} bh\) **Area of Triangle**
\[
\frac{dA}{dt} = \frac{1}{2} \left( b \cdot \frac{dh}{dt} + h \cdot \frac{db}{dt} \right)
\]

4. \(\tan \theta = \frac{v}{x}\) \(\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt}}{x^2} \)

5. \(V = e^t\) \((e\) stands for edges\).
\[
\frac{dV}{dt} = 3e^t \cdot \frac{de}{dt}
\]

6. \(S.A. = 6e^t\)**SA of Cube**
\[
\frac{dSA}{dt} = 6 \cdot 2e \cdot \frac{de}{dt} = 12e \cdot \frac{de}{dt}
\]

7. \(V = \frac{1}{3} \pi \cdot r^2 \cdot h\) **Vol. of Cone**
\[
\frac{dV}{dt} = \frac{1}{3} \pi \cdot \left( r^2 \cdot \frac{dh}{dt} + h \cdot 2r \cdot \frac{dr}{dt} \right)
\]

8. \(V = \frac{1}{3} \pi \cdot r^2 \cdot h\), where \(r = \frac{h}{3}\)
(supstitute for \(r\) before differentiating)
\[
V = \frac{1}{3} \pi \left( \frac{h}{3} \right)^2 \cdot h = \frac{\pi h^3}{27}
\]
\[
\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi h^2}{9} \cdot \frac{dh}{dt}
\]

9. “Related rates” problems are always differentiated with respect to **Time**_____

Name ___________________________
10. Ladder problem - (see p. 249)

(a) Why is \( \frac{dx}{dt} \) positive?

(b) Why is \( \frac{dy}{dt} \) negative?

(c) How do you find \( \frac{dA}{dt} \)?

(d) Describe the sign possibilities for \( \frac{dA}{dt} \).

\[
\frac{dA}{dt} = \frac{1}{2} \left( x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt} \right)
\]

11. How would you set up (and then differentiate) the volume of a cone if the ratio of radius to height is 2:5 and you want to find \( \frac{dh}{dt} \)?

\[
V = \frac{1}{3} \pi r^2 h
\]

\[
V = \frac{4 \pi}{3} \left( \frac{2h}{5} \right)^2 \cdot h
\]

\[
V = \frac{4 \pi h^3}{75}
\]

\[
\frac{dV}{dt} = \frac{4 \pi}{75} \cdot 3 h^2 \cdot \frac{dh}{dt}
\]

12. An airplane is flying at an altitude of 6 miles on a path that will take it directly over a radar tracking station. If the distance to the station is decreasing at a rate of 400 miles per hour when the distance is 10 miles, what is the speed of the plane?

\[
\frac{dx}{dt} = -400 \text{ mi/hr}
\]

\[
\frac{dZ}{dt} = -500
\]

\[
\text{Speed of plane is 500 mph.}
\]