A. These all mean (basically) the same thing:

1. Slope of the tangent line
2. Slope of the curve
3. Instantaneous Rate of Change
4. \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

B. A normal line is a line perpendicular to the tangent line at point of tangency.

C. \( \frac{f(a+h) - f(a)}{h} \) is called a difference quotient.

D. Symmetric difference quotient:

\[
\frac{f(a+h) - f(a-h)}{2h}
\]

Definition of Derivative - \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) (Say "f-prime of x.")

One of the applications of derivative is to find the slope of (the tangent line to) a curve.

Ex - Find the slope of the tangent line to \( y = x^2 - 3x - 1 \) at the point \( x = 0 \) using two different methods.

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
f'(0) = \lim_{h \to 0} \frac{0^2 - 3(0) - 1 - (x^2 - 3x - 1)}{h}
\]

\[
= \lim_{h \to 0} \frac{2xh + h^2 - 3h - 1 - x^2 + 3x + 1}{h}
\]

\[
= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}
\]

\[
= \lim_{h \to 0} (2x + h - 3)
\]

\[
f'(0) = 2(0) - 3 = -3
\]

\[
f'(0) = -3
\]
Find the derivative of each of the following:

1. \( f(x) = 4x^2 - 3x - 5 \)
   
   \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
   
   \[ = \lim_{h \to 0} \frac{4(x+h)^2 - 3(x+h) - 5 - (4x^2 - 3x - 5)}{h} \]
   
   \[ = \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 3x - 3h - 5 - 4x^2 + 3x + 5}{h} \]
   
   \[ = \lim_{h \to 0} \frac{8xh + 4h^2 - 3h}{h} = \lim_{h \to 0} \frac{(8x + 4h - 3)}{h} \]
   
   \[ = 8x + 4h - 3 \]
   
   \[ \Rightarrow f'(x) = 8x - 3 \]

2. \( f(x) = \frac{1}{2-x} \)
   
   \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
   
   \[ = \lim_{h \to 0} \frac{1}{2-(x+h)} - \frac{1}{2-x} \]
   
   \[ = \lim_{h \to 0} \frac{1}{2-x-h} \left( \frac{2-x}{2-x-h} \right) \]
   
   \[ = \lim_{h \to 0} \frac{2-x - (z-x-h)}{h (2-x)(2-x-h)} \]
   
   \[ = \lim_{h \to 0} \frac{2-x - z + x + h}{h (2-x)(2-x-h)} \]
   
   \[ = \lim_{h \to 0} \frac{(2-x)(2-x-h)}{(2-x)(2-x-h)} = \frac{1}{(2-x)(2-x)} \]
   
   \[ \Rightarrow f'(x) = \frac{1}{(2-x)^2} \]

3. \( f(x) = \sqrt{x+3} \)
   
   \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
   
   \[ = \lim_{h \to 0} \frac{\sqrt{(x+h)+3} - \sqrt{x+3}}{h} \]
   
   \[ = \lim_{h \to 0} \frac{(\sqrt{(x+h)+3} - \sqrt{x+3})(\sqrt{(x+h)+3} + \sqrt{x+3})}{h(\sqrt{(x+h)+3} + \sqrt{x+3})} \]
   
   \[ = \lim_{h \to 0} \frac{x+h+3 - (x+3)}{h(\sqrt{(x+h)+3} + \sqrt{x+3})} \]
   
   \[ = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + 3 + \sqrt{x+3})} \]
   
   \[ = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + 3 + \sqrt{x+3}} \]
   
   \[ = \frac{1}{\sqrt{x+3} + \sqrt{x+3}} \]
   
   \[ \Rightarrow f'(x) = \frac{1}{2\sqrt{x+3}} \]