Intermediate Value Theorem – A function \( f(x) \) that is continuous on \((a, b)\) takes on all values between \( f(a) \) and \( f(b) \).

Average Rate of Change – same as the slope of the secant line.

\[
\text{avg rate of change} = \frac{f(b) - f(a)}{b - a}
\]

ex – Find the average rate of change of \( f(x) = x^2 + 2x + 1 \) on \([0, 3]\).

\[
\begin{align*}
f(3) &= 3^2 + 2(3) + 1 = 9 + 6 + 1 = 16 \\
f(0) &= 0^2 + 2(0) + 1 = 1
\end{align*}
\]

Look at this slightly differently. Suppose we are interested in the rate of change at a particular moment.

Instantaneous rate of change =

\[
\lim_{{h \to 0}} \frac{f(x+h) - f(x)}{x+h - x} = \lim_{{h \to 0}} \frac{f(x+h) - f(x)}{h}
\]

On a curve, the instantaneous rate of change is the slope of the tangent line.
ex – Find the slope of the “curve” \( f(x) = 2x - 3 \) at \( x = a \).

\[
\text{slope} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
= \lim_{h \to 0} \frac{2(a+h) - 3 - (2a-3)}{h} \\
= \lim_{h \to 0} \frac{2a + 2h - 3 - 2a + 3}{h} = \lim_{h \to 0} \frac{2h}{h} \\
= \lim_{h \to 0} 2 = 2
\]

ex – Find the slope of the curve \( f(x) = \frac{2}{x} \) at the point \( x = 3 \) \( (a = 3) \). (Note: This question could also be written as “find the slope of the tangent line to the curve . . . . ”)

\[
\text{slope} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} \\
= \lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h} \\
= \lim_{h \to 0} \frac{\frac{2}{3+h} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{3}{(3+h)}}{h} \\
= \lim_{h \to 0} \frac{6 - 2(3+h)}{3h(3+h)} = \lim_{h \to 0} \frac{6 - 6 - 2h}{3h(3+h)} = \lim_{h \to 0} \frac{-2h}{3h(3+h)} \\
= \frac{-2}{3(3+0)} = \frac{-2}{9}
\]