AP Calculus – 30 Points

Name: ___________________________ Date: ______________

MIDTERM REVIEW QUIZ 1. Take Home. No Internet. No Friends. You may use your notes and your calculator. Must have appropriate work and/or “calculator talk” to get the correct answer. Be prepared for me to call on you to do a problem for the class if you get the problem correct on the quiz.

1) What is the slope of the curve \( y = 3\sin x - 2 \) at its first positive \( x \)-intercept?
   (A) 0.683
   (B) 1.643
   (C) 1.705
   (D) 1.805
   (E) 2

   \[ y'_{(x_0)} = \frac{d}{dx} (3\sin x - 2) \bigg|_{x = x_0} = 3\cos x \]
   \( x_0 = \arcsin \left( \frac{2}{3} \right) \approx 0.644 

2) The wind chill is the temperature, in degrees Fahrenheit (\( ^\circ F \)), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity \( v \), in miles per hour (mph). If the air temperature is \( 32^\circ F \), then the wind chill is given by \( W(v) = 55.6 - 22.1v^{0.16} \) and is valid for \( 5 \leq v \leq 60 \).

   Find \( W'(20) \). Using correct units, explain the meaning of \( W'(20) \) in terms of wind chill.

   \[ W'(v) = \frac{d}{dv} (55.6 - 22.1v^{0.16}) = -22.1 \cdot 0.16v^{-0.84} \]
   \[ W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} \approx -0.286 \text{ \textdegree F/mph} \]

3) Find the average rate of change of \( W \) over the interval \( 5 \leq v \leq 60 \). Find the value of \( v \) at which the instantaneous rate of change of \( W \) is equal to the average rate of change of \( W \) over the interval \( 5 \leq v \leq 60 \).

   \[ \frac{W(60) - W(5)}{60 - 5} = \frac{55.6 - 22.1(60)^{0.16} - (55.6 - 22.1(5)^{0.16})}{55} \approx -0.253 \text{ \textdegree F/mph} \]

4) Let \( f \) be the function given by \( f(x) = 2e^{4x^2} \). For what value of \( x \) is the slope of the line tangent to the graph of \( f \) at \( (x, f(x)) \) equal to 3?

   \[ y = \frac{d}{dx} (2e^{4x^2}) = 8xe^{4x^2} \]
   \[ 3 = 8xe^{4x^2} \]
   \[ x = \frac{\ln(3/8)}{4} \approx 0.675 \]

5) The first derivative of the function \( f \) is defined by \( f'(x) = \sin(x^3 - x) \) for \( 0 \leq x \leq 2 \). On what interval(s) is \( f \) increasing?

   (A) \( 1 \leq x \leq 1.445 \)
   (B) \( 1 \leq x \leq 1.691 \)
   (C) \( 1.445 \leq x \leq 1.875 \)
   (D) \( 0.577 \leq x \leq 1.445 \) and \( 1.875 \leq x \leq 2 \)
   (E) \( 0 \leq x \leq 1 \) and \( 1.691 \leq x \leq 2 \)

   \[ f'(x) = \cos(x^3 - x) \cdot (3x^2 - 1) \]

   \[ f'(x) > 0 \text{ for all } x \in (a, b), \]
   then \( f \) is increasing on \( (a, b) \)

   \[ y_1 = \sin(x^3 - x) \]
   \[ \text{set window to } x = 0 \text{ to } x = 2 \]

   \[ (1, 1.691) \]
6) If \( f'(x) = \sqrt{x^4 + 1} + x^3 - 3x \), then \( f \) has a local maximum at \( x = \) 
- (A) -2.314 
- (B) -1.332 
- (C) 0.350 
- (D) 0.829 
- (E) 1.234 

\[ y_1 = (x^4 + 1)^{\frac{1}{2}} + x^3 - 3x \]

\[ y_1(x) = 0, x \]

\[ f \text{ has a max at } x = 0.350 \]

b/c the sign of \( f'(x) \) changes from (+) to (-) 
\[ a + x = 0.350 \]

Graph 

\[ x = -2.314 \]
\[ x = 0.350 \]
\[ x = 1.234 \]

7) For \( 0 \leq t \leq 2\pi \), let \( f'(x) = e^{\sin(2x)} \). On which of the following intervals is \( f \) concave down?

- (A) \((0.333, 1.238) \cup (3.475, 4.379)\) 
- (B) \((0.0, 0.333) \cup (1.238, 3.475) \cup (4.379, 6.283)\) 
- (C) \((0.785, 3.927, 6.283)\) 
- (D) \((0.785, 2.356) \cup (3.927, 5.498)\) 
- (E) \((0.785, 2.356) \cup (3.927, 5.498)\)

\[ y_1 = e^{\sin(2x)} \]
\[ y_1 = y(\sin(2x)) \]
\[ y_1 = d(y_1, (x), x) \]
\[ y_2 = d(y_1, (x), x) \]

\[ 0 \leq t \leq 2\pi \]

\[ \text{to uncheck } y_1 \text{ & only graph } y_2 \]

8) A particle moves along a straight line with velocity given by \( v(t) = 7 - (1.01)^{-t} \) at time \( t \geq 0 \). What is the acceleration of the particle at time \( t = 3 \)? (2008 Exam)

- (A) -0.914 
- (B) 0.055 
- (C) 5.486 
- (D) 6.086 
- (E) 18.087 

\[ v'(3) = a(3) = 0.0546 \]

9) The velocity, in ft/sec, of a particle moving along the x-axis is given by the function \( v(t) = e^t + te^t \).

What is the average velocity from the time \( t = 0 \) to \( t = 3 \) ?

- (A) 20.086 ft/sec 
- (B) 26.447 ft/sec 
- (C) 32.809 ft/sec 
- (D) 40.671 ft/sec 
- (E) 79.342 ft/sec 

\[ \frac{dy}{dx} = \frac{-3x^2 + y}{3y^2 - x} \]
\[ \frac{dy}{dx} = \frac{y - 3}{3y^2 - 1} \]

average velocity = 26.447 ft/sec

10) What is the approximate slope of the tangent to the curve \( x^3 + y^3 = xy \) at \( x = 1 \)?

- (A) -2.420 
- (B) -1.325 
- (C) 1.014 
- (D) -0.698 
- (E) 0.267 

\[ x^3 + y^3 = xy \]
\[ 1 + y^3 = 3y \]
\[ (1 + x^3 = y, x) \]
\[ -1.3247179 \to y \]
\[ 1 \to 1 \]

\[ \frac{dy}{dx} = \frac{y - 3}{3y^2 - 1} \]
\[ \frac{dy}{dx} = \frac{y - 3}{3y^2 - 1} \]

\[ \frac{dy}{dx} \bigg|_{y = -1.3247} = \frac{y - 3}{3y^2 - 1} \]

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